H16 年度微分積分演習　No.3　('04/4/20)　学籍番号　　　　　　　氏名

1.　By taking logarithms of the both sides, find the derivative for $y = x^x$ ($x > 0$).

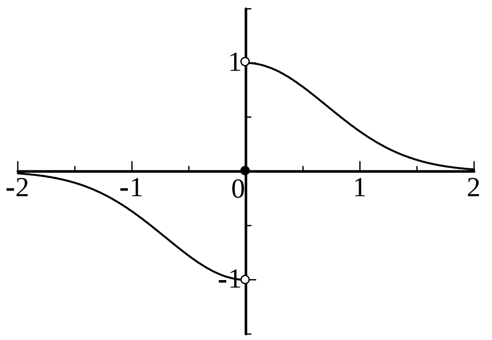
$$\log y = \log x^x$$

$$\frac{d}{dy}(\log y) = \frac{y'}{y} \qquad \frac{d}{dx}(\log x^x) = \frac{d}{dx}(x \log x) = 1 \cdot \log x + x \cdot \frac{1}{x} = \log x + 1$$

$$\therefore \frac{y'}{y} = \log x + 1 \qquad \therefore y' = y(\log x + 1) = x^x(\log x + 1)$$

2.

$$f(x) = \begin{cases} e^{-x^2} & (x > 0) \\ 0 & (x = 0) \\ -e^{-x^2} & (x < 0) \end{cases}$$



i. Calculate the limitting values of $\lim_{x \to +0} f'(x)$ and $\lim_{x \to -0} f'(x)$.

ii. Say $f(x)$ is differential at $x = 0$ or not.

i. Since the functions of $g(x) = x^2$, $h(x) = e^x$ and $s(x) = -x$ are differentiable for $x \neq 0$, the function $f(x)$ constituted by them is also differentiable for $x \neq 0$.

$$f'(x) = e^{-x^2} \cdot (-2x) \to 0 \qquad (x \to \pm 0)$$

ii. At $x = 0$,

$$\lim_{h \to +0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to +0} \frac{e^{-h^2} - 0}{h} = +\infty \qquad \lim_{h \to -0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to -0} \frac{e^{-h^2} - 0}{h} = -\infty$$

The slope of a curve does not exist at $x = 0$, i.e. $f(x)$ is not differentiable at $x = 0$.

Both of the limitting values of $\lim_{x \to +0} f'(x)$ and $\lim_{x \to -0} f'(x)$ are zero and coincident with each other.

However, the slope of the curve does not exist at $x = 0$ since $f(x)$ is **discontinuous at** $x = 0$.